USE OF A THREE-LAYER DISTRIBUTED RC NETWORK

TO PRODUCE TWO PAIRS OF COMPLEX CONJUGATE ZEROS

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Abstract: This report describes the properties of a three-layer
distributed RC network consisting of two layers of resistive
material separated by a dielectric. When the three-layer network
is used as a three-terminal element by connecting conducting
terminal strips across the ends of one of the resistive layers
and the center of the other resistive layer, the network may be
used to produce pairs of complex conjugate transmission zeros.
The locations of these zeros are determined by the parameters
of the network. Design charts for determining the zero positions
are included as part of the report.

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I. Introduction

This is one of a series of reports describing the use of digital

computational techniques in the analysis and synthesis of DLA

(Distributed-Lumped-Active) networks. This class of networks

consists of three distinct types of elements, namely, distributed

elements (modeled by partial differential equations), lumped

elements (modeled by algebraic relations and ordinary differential

equations), and active elements (modeled by algebraic relations).

Such a characterization is applicable to a broad class of

circuits, especially including those usually referred to as linear

integrated circuits, since the fabrication techniques for such

circuits readily produce elements which may be modeled as

distributed, as well as the more conventional lumped and active

ones. The network functions which describe distributed elements,

however, involve hyperbolic irrational functions of the complex

frequency variable. The complexity of such functions make the

use of digital computational techniques most desirable in analyzing

and synthesizing these networks.

In this report we shall consider the application of such

digital computational techniques to the analysis and synthesis of

a three-layer distributed RC network element. This element was

first shown in a previous report to have the capabilities of

producing a pair of transmission zeros on the $j\omega$ axis.[1] In this report we shall extend the previous results to show that the transmission zeros produced by the three-layer network may be located anywhere in the complex frequency plane. In addition, a series of design charts is given showing the locations of those zeros.

II. The Three-Layer Distributed RC Network

A three-layer uniform distributed-RC network, as analyzed in this report, consists of two layers of resistive material separated by a layer of dielectric material. As shown in Fig. 1, such a network may be considered as a three-terminal network element by adding three conducting terminal strips, two across the ends of one of the resistive layers, and one across the mid-region of the other resistive layer. The network may be characterized schematically as shown in Fig. 2, in which R is the total resistance of the resistive layer with two terminals, NR (where N is a positive constant of proportionality) is the total resistance that is measured between the ends of the resistive layer with the single terminal (if terminal strips were provided at the ends), and C is the total capacitance that exists between the two resistive layers. Because of the assumption that the distributed resistance and capacitance are uniform, i.e., R and C are constant rather than being functions of position, the network shown in Fig. 2 may be modeled as a cascade connection of two uniform distributed RC networks connected in cascade as shown in Fig. 3. In this figure, the
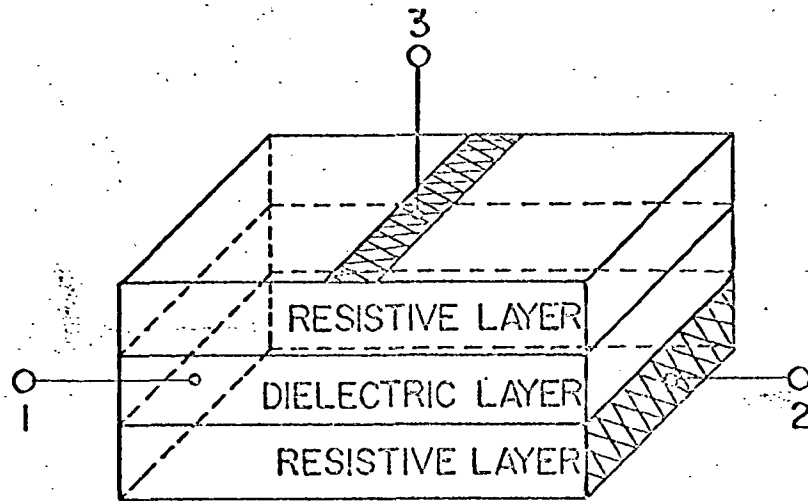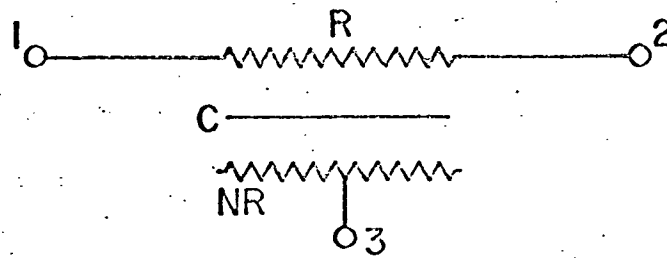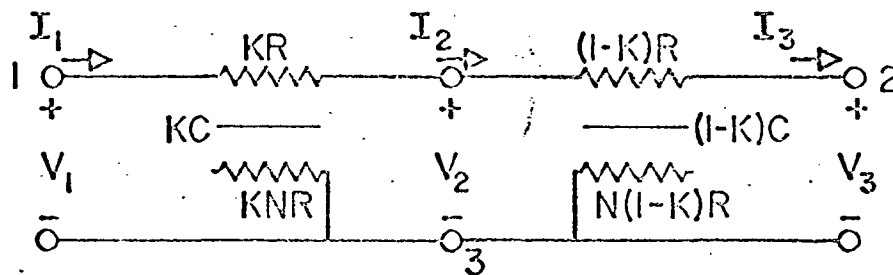
FIG. 1

FIG. 2

FIG. 3

constant K has been defined to indicate the relative position of the terminal strip on the single-terminal resistive layer. For example, for K equals zero, the terminal strip is at the left end of the network shown in Fig. 1, and for K equals unity, it is at the right end. The voltage transfer function for this network may be shown to be[1]

$$\frac{V_2}{V_1} = \frac{\frac{\sinh K\theta}{\theta}(N\cosh K\theta + 1)\quad (N\cosh M\theta + 1)}{\frac{\sinh \theta}{\theta}\left[NK\theta^2 \frac{\sinh K\theta}{\theta} + (N^2+1)\cosh K\theta + 2N\right] - \frac{\sinh M\theta}{\theta}(N\cosh K\theta + 1)^2} \qquad (1)$$

where $\theta = \sqrt{pRC(N+1)}$, p is the complex frequency variable, and M=1-K. If N is set to the value 0.086266738, then transmission zeros are produced on the $J\omega$ axis, the relative locations of these zeros being determined by the value of K.

III. Complex Conjugate Transmission Zeros

Now let us consider the effect of using values of N other than the one specified in the preceding section. Choosing a range of values of N from 0.01 to 0.5 and a range of values of K from 0.05 to 0.5, and applying digital computational techniques similar to those described in the previous report, we obtain the charts shown in Figs. 4 and 5. Specifically Fig. 4 shows upper half-plane location of the lower complex conjugate transmission zero of the network as a function of K and N, and Fig. 5 shows the upper half-plane location of the upper complex conjugate zero. Different scales are used for the two figures, however, the curve for K=0.5,
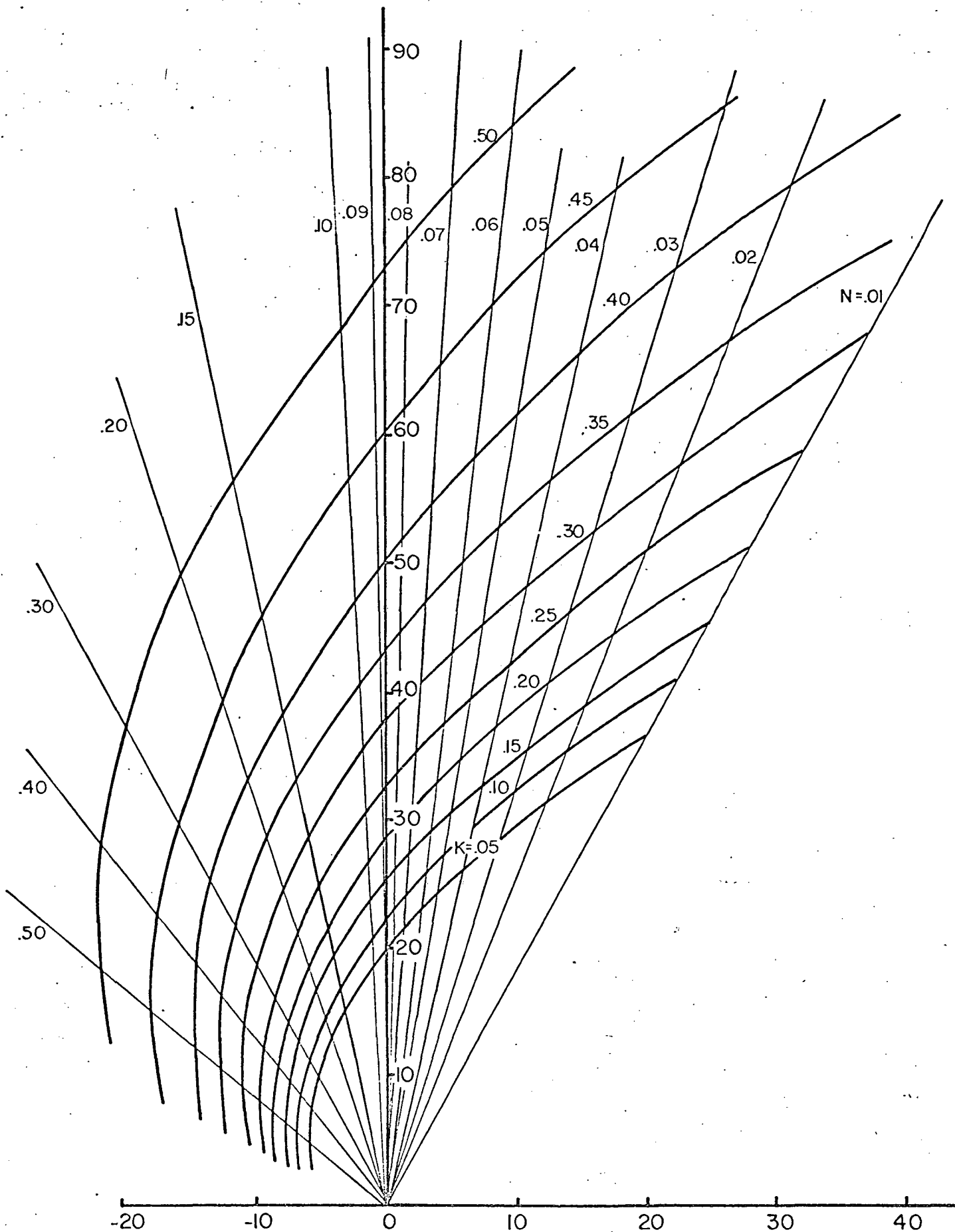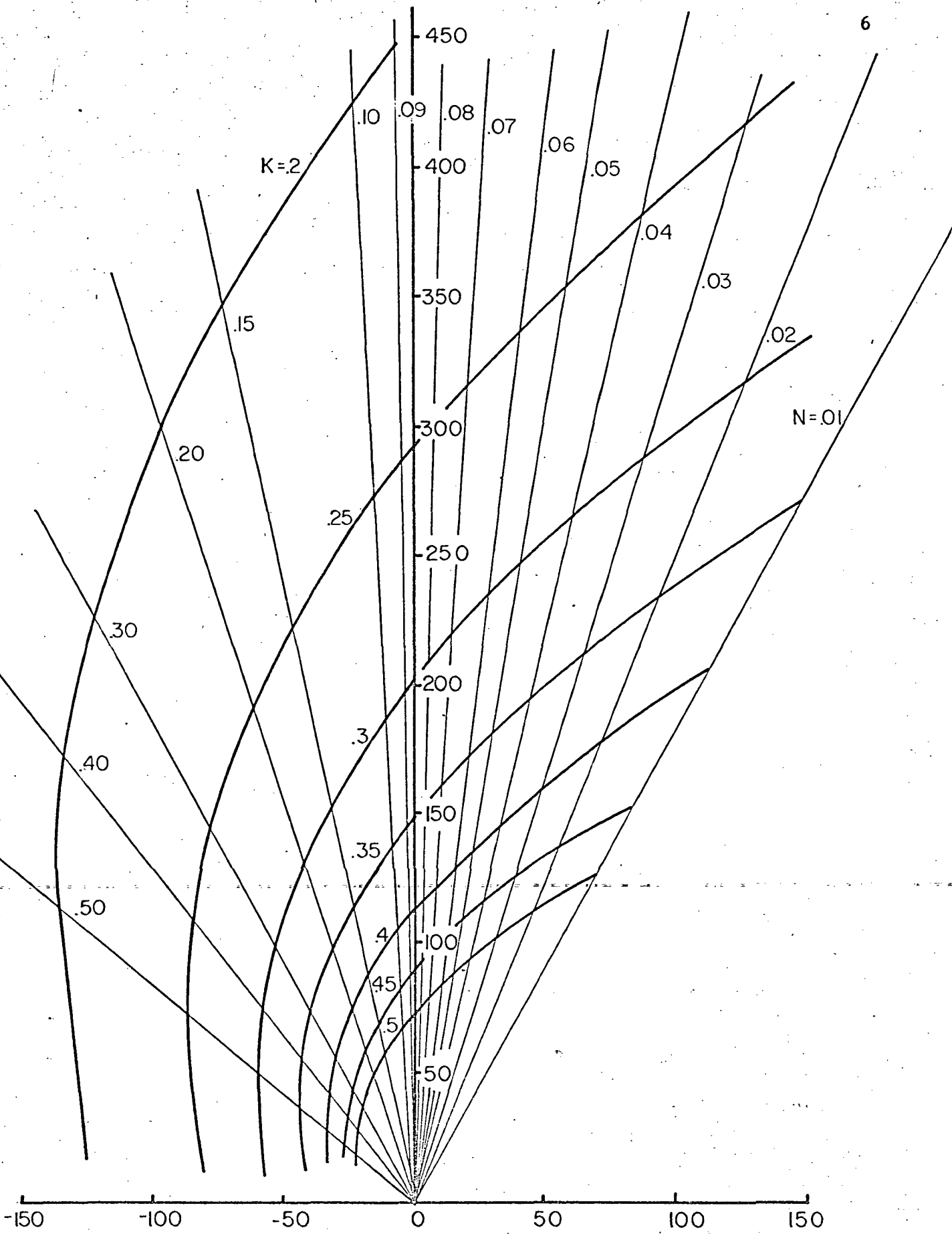
Fig. 4

Fig. 5

representing the condition when the two complex-conjugate zeros are coincident and thus form a pair of second-order complex conjugate zeros, is shown on both figures to provide a common reference.

IV. Applications

There are two main applications which may be made of the complex conjugate zeros produced by the three-layer distributed RC network. First of all, we note that for K=0.5, a second-order pair of dominant complex conjugate zeros are produced. Thus, for such a value of K, this network is capable of producing rejection characteristics which are more sharply defined than networks which have only a first order dominant complex conjugate transmission zero, such as the Kaufmann and Garrett lump-distributed RC notch network.[2] To illustrate this in Fig. 6, we show reciprocal sinusoidal steady-state characteristics for the transmission curves for the three-layer network (K=0.5) and the Kaufman and Garrett network. The 4 rad/sec and 8 rad/sec bandwidths for each of these networks is shown on the plot. For the three-layer network, there is approximately 10.5 dB between these two bandwidths, while for the Kaufman and Garrett network there is only about 5 dB. The improvement in selectivity made by the second-order zeros of the three-layer network is clearly evident. A second application which may be made for the three-layer distributed RC network is as a pole-determining element in a feedback loop around some active device.
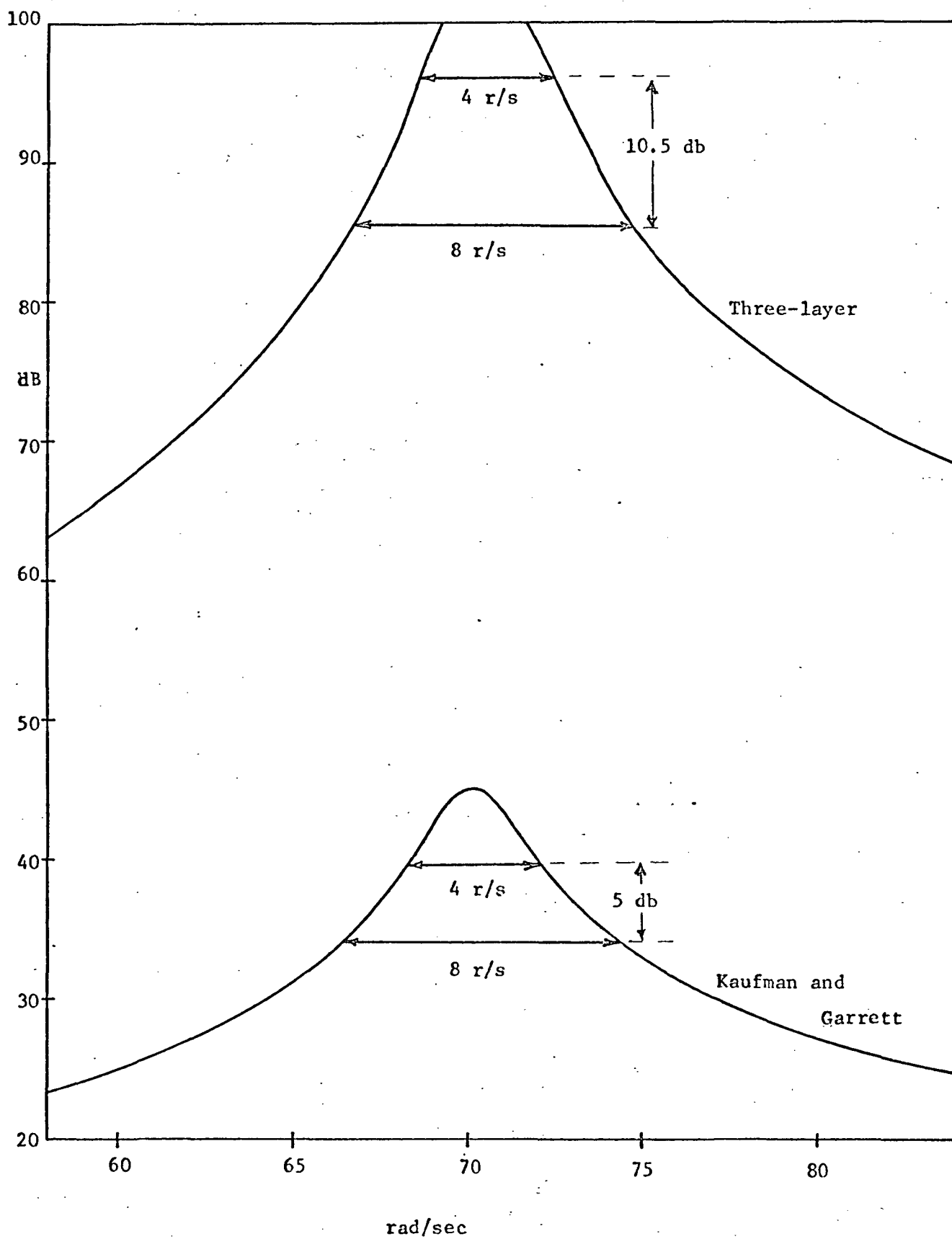
Fig. 6

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In such an application the transmission zeros of the distributed RC network become the poles of the overall network transfer function. As an example of the potentialities of the distributed RC network in producing a broad-band amplifier, response curves of the reciprocal of the voltage transfer function for $N = 0.1$, and for a range of values of K are shown in Fig. 7. The broadening of the resonant peak due to the separation of the complex-conjugate zeros of transmission is readily evident.

V. Conclusion

In this report we have indicated the potential of the three-layer distributed RC network for producing complex-conjugate transmission zeros. The use of other degrees of freedom of the network in determining the location of such zeros, such as appropriately tapering the distributed RC element, is currently under study.
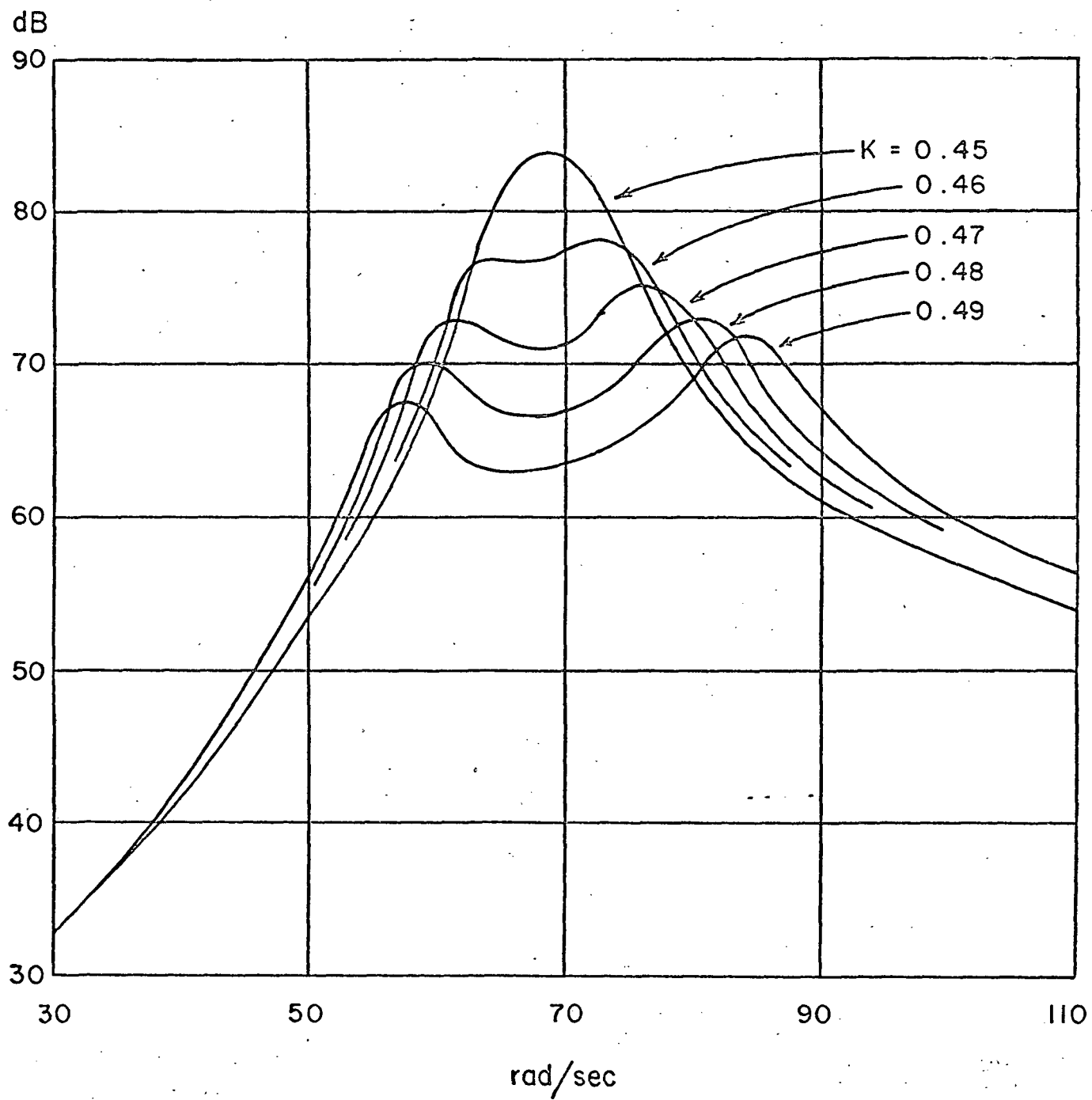
Acknowledgments

Fig. 7

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